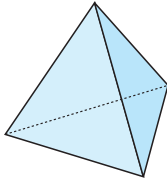


INVESTIGATION

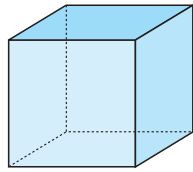
PLATONIC SOLIDS

Platonic solids are regular polyhedra whose faces are all the same shape.

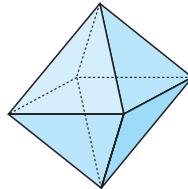
There are exactly five platonic solids: the tetrahedron, cube, octahedron, dodecahedron and icosahedron.



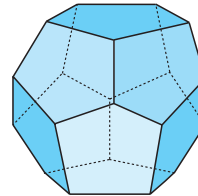
Tetrahedron



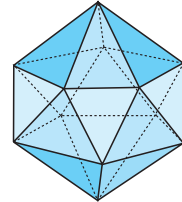
Cube



Octahedron



Dodecahedron



Icosahedron

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ing Euler's formula and

ners (vertices), e edges,

the degrees be p where $p \geq 3$. Also, each region of the graph has the same shape, so we let the number of sides in each region be q , where $q \geq 3$.

Then $pv = qf = 2e$.

Also, since the solids are planar, we have Euler's relation $v - e + f = 2$.

- 1** Show that $8 = v(4 - p) + f(4 - q)$.

Since v and f are both positive, $(4 - p)$ and $(4 - q)$ cannot both be negative.

$\therefore$ either $4 - p \geq 0$ or $4 - q \geq 0$

$\therefore 3 \leq p \leq 4$ or $3 \leq q \leq 4$, though not necessarily both together.

We now have four cases to investigate: $p = 3$, $p = 4$, $q = 3$ and $q = 4$ and we must consider each of these in turn to complete our proof.

- 2** For the case $p = 3$, use $qf = 3v = 2e$ and the modified Euler relation derived in **1** to show that $f(6 - q) = 12$, $f, q \in \mathbb{Z}^+$.

Using the factors of 12, we can consider the separate cases from the equation derived in **2**:

- $f = 1 \quad \therefore q < 0$ which is invalid
- $f = 2 \quad \therefore q = 0$ which is invalid
{a region without edges}
- $f = 3 \quad \therefore q = 2 \Rightarrow v = 2$ which is invalid
{3 vertices required for a region}
- $f = 4 \quad \therefore q = 3 \Rightarrow v = 4, e = 6$ a tetrahedron
- $f = 6 \quad \therefore q = 4 \Rightarrow v = 8, e = 12$ a cube
- $f = 12 \quad \therefore q = 5 \Rightarrow v = 20, e = 30$ a dodecahedron

- 3** For the case $p = 4$, use $qf = 4v = 2e$ and the modified Euler relation to show that $f(4 - q) = 8$, $f, q \in \mathbb{Z}^+$. Use the procedure above and hence show that an octahedron is a platonic solid.

- 4** Repeat **3** for the cases $q = 3$ and $q = 4$.

Having considered all cases, you should have proven there are exactly five platonic solids.

Questions:

- 1** Draw a Schlegel diagram for each platonic solid.
- 2** Draw a Hamiltonian path on each diagram drawn in **1**.